Lesson 37 Imaginary and Complex Numbers

Solve each equation for x.

1. $x - 1 = 0$

   +1 +1

   x = 1

2. $x + 1 = 0$

   -1 -1

   x = -1

3. $x^2 - 1 = 0$

   +1 +1

   $x^2 = 1$

   $\sqrt{x^2} = \pm\sqrt{1}$

   $x = \pm 1$

4. $x^2 + 1 = 0$

   -1 -1

   $x^2 = -1$

   $\sqrt{x^2} = \pm\sqrt{-1}$

   $x = \pm i$

Imaginary Numbers – numbers that involve the square root of negative 1.
Examples: $2i, -6i, 10i$

Complex Numbers – have a real part and an imaginary part
Examples: $3 + 4i, -2 - 7i, 0 + 3i, -8 - i, -5 + 0i$

Addition and Subtraction with Complex Numbers – Combine like terms. You do not need to write out each step as shown below in numbers 5 and 6, if you can go from the problem to the final answer like in number 7.

5. $(3 + 4i) + (7 - 20i)$

$$(3 + 4i) + (7 - 20i) = 3 + 4i + 7 - 20i = (3 + 7) + (4 - 20)i = 10 - 16i$$

6. $(3 + 4i) - (7 - 20i)$

$$(3 + 4i) - (7 - 20i) = 3 + 4i - 7 + 20i = (3 - 7) + (4 + 20)i = -4 + 20i$$

7. $(6 - i) + (3 - 2i) = 9 - 3i$

Multiplication with Complex Numbers – FOIL, distribut, or use a Punnett Square

$i = \sqrt{-1}$
$i^2 = (\sqrt{-1})^2$ If we square both sides,
$i^2 = -1$ we get this.

You will need to use $i^2 = -1$ to finish simplifying. Replace i^2 with -1.

8. $(\mathbf{1 + 2}\boldsymbol{i})(\mathbf{1 - 2}\boldsymbol{i})$

OR

	1	+2i
1	1	2i
-2i	-2i	$-4i^2=-4(-1)=-4$

$= \mathbf{1 + 2}\boldsymbol{i} \mathbf{- 2}\boldsymbol{i} \mathbf{- 4}\boldsymbol{i}^{\mathbf{2}}$
$= \mathbf{1 + 0 - 4(-1)}$
$= \mathbf{1 + 4}$
$= \mathbf{5}$

9. $(3 + 2i)(-3 + 2i) = -13$
$= -9 + 6i - 6i + 4i^2$
$= -9 + 4(-1)$
$= -9 - 4$
$= -13$

10. $(5 + 4i)(2 - i) = 14 + 3i$
$= 10 - 5i + 8i - 4i^2$
$= 10 + 3i - 4i^2$
$= 10 \mp 3i - 4(-1)$
$= 10 + 3i + 4$
$= 14 + 3i$

11. $(1 - 6i)^2$
$= (1 - 6i)(1 - 6i)$
$= 1 - 6i - 6i + 36i^2$
$= 1 - 12i + 36(-1)$
$= 1 - 12i - 36$
$= -35 - 12i$

Simplifying Powers of i – these can all be simplified to $i, -1, -i, or\ 1$

$i = \sqrt{-1}$
$i^2 = -1$
$i^3 = i^2 \cdot i = (-1) \cdot i = -i$
$i^4 = i^2 \cdot i^2 = (-1) \cdot (-1) = 1$

So, to simplify higher powers of i, break them down into as many i^4 as you can. Those are equal to 1. Then simplify what is left over as done above.

12. $i^{37} = i^4 \cdot i^4 \cdot i^4 \cdot i^4 \cdot i^4 \cdot i^4 \cdot i^4 \cdot i^4 \cdot i^4 \cdot i = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot i = i$

When multiplying the same base (i), you add the exponents. So, the exponents add to 37. The i^4 can all be replaced with 1. There is only one i left, so this simplifies to i.

Here is another way to do this. Find a multiple of 4 (4, 8, 12, 16, 20, 24, 28, 32, 36, 40,…) that is close to the exponent of 37 without going over. Write i to that power(36) times i to a power (1) so you get the number of i that you are simplifying. Rewrite i^4 to a power, so the exponents multiply to the multiple of 4. All of the i^4 simplify to 1.

$$i^{37} = i^{36} \cdot i = (i^4)^9 \cdot i = 1^9 \cdot i = i$$

13. $i^{30} = (i^4)^7 \cdot i^2 = 1 \cdot (-1) = -1$

14. $i^{43} = (i^4)^{10} \cdot i^3 = 1 \cdot i = -i$

15. $i^{60} = (i^4)^{15} = 1$