

Average Value of a Function

How have we found Average Velocity?

How have we found Average Acceleration?

If $p(t)$, $v(t)$, and $a(t)$ represent position, velocity and acceleration defined for any time t , write an equivalent expression for each of the following integrals based on the fundamental theorem of calculus.

$\frac{1}{b-a} \int_a^b a(t) dt =$		To what is this equivalent?
$\frac{1}{b-a} \int_a^b v(t) dt =$		To what is this equivalent?

The average value of a function, $f(x)$, on an interval $[a, b]$ is defined to be:

Find the average value of the function $f(x) = x^3 \sqrt{\sin^2 x}$ on the interval $1 \leq x \leq 3$. [Calculator]

Find the average value of the function $f(x) = 2 - 4x$ on the interval $2 \leq x \leq 6$. [Noncalculator]

A ski resort uses a snow machine to control the snow level on a ski slope. Over a 24-hour period the volume of snow added to the slope per hour is modeled by the equation $S(t) = 24 - t \sin^2\left(\frac{t}{14}\right)$. The rate at which the snow melts is modeled by the equation $M(t) = 10 + 8 \cos\left(\frac{t}{3}\right)$. Both $S(t)$ and $M(t)$ have units of cubic yards per hour and t is measured in hours for $0 \leq t \leq 24$. At time $t = 0$, the slope holds 50 cubic yards of snow.

a. Compute the total volume of snow added to the mountain over the first 6-hour period.

b. Find the value of $\int_0^6 M(t) dt$ and $\frac{1}{6} \int_0^6 M(t) dt$. Using correct units of measure, explain what each represents in the context of this problem.

c. Is the volume of snow increasing or decreasing at time $t = 4$? Justify your answer.

d. How much snow is on the slope after 5 hours? Show your work.

e. Suppose the snow machine is turned off at time $t = 10$. Write, but do not solve, an equation that could be solved to find the time $t = K$ when the snow would all be melted.

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Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5-x & \text{for } 3 < x \leq 5. \end{cases}$$

(b) Find the average value of $f(x)$ on the closed interval $0 \leq x \leq 5$.

