

Day #53 Homework

At time $t = 0$, there are 120 pounds of sand in a conical tank. Sand is being added to the tank at the rate of $S(t) = 2e^{\sin^2 t} + 2$ pounds per hour. Sand from the tank is used at a rate of $R(t) = 5\sin^2 t + 3\sqrt{t}$ per hour. The tank can hold a maximum of 200 pounds of sand.

1. Find the value of $\int_0^4 S(t) dt$. Using correct units, what does this value represent?
2. Find the value of $\int_1^3 R(t) dt$. Using correct units, what does this value represent?
3. Find the value of $\frac{1}{4} \int_0^4 S(t) dt$. Using correct units, what does this value represent?
4. Write a function, $A(t)$, containing an integral expression that represents the amount of sand in the tank at any given time, t .
5. How many pounds of sand are in the tank at time $t = 7$?
6. After time $t = 7$, sand is not used any more. Sand is, however, added until the tank is full. If k represents the value of t at which the tank is at maximum capacity, write, but do not solve, an equation using an integral expression to find how many hours it will take before the tank is completely full of sand.

2005 AP[®] CALCULUS AB

Problem #2

The tide removes sand from Sandy Point Beach at a rate modeled by the function R , given by

$$R(t) = 2 + 5 \sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S , given by

$$S(t) = \frac{15t}{1 + 3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \leq t \leq 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

- How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.
- Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t .
- Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.
- For $0 \leq t \leq 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

2005 AP[®] CALCULUS AB

Problem #3

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ ($^{\circ}\text{C}$)	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius ($^{\circ}\text{C}$), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

- (a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.
- (b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.
- (c) Find $\int_0^8 T'(x) dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x) dx$ in terms of the temperature of the wire.
- (d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

2002 AP[®] CALCULUS AB (Form B)

Problem #2

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

- (a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?
- (b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.
- (c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.
- (d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

2000 AP[®] CALCULUS AB

Problem #4

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \leq t \leq 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

- (a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?
- (b) How many gallons of water are in the tank at time $t = 3$ minutes?
- (c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t .
- (d) At what time t , for $0 \leq t \leq 120$, is the amount of water in the tank a maximum? Justify your answer.

