Integration of Composite Functions

For each of the functions given below, find both $f'(x)$ and $\int f'(x)dx$.

$f(x)$	$f'(x)$	$\int f'(x)dx$
$f(x) = \sin 3x$		
$f(x) = e^{\cos x}$		
$f(x) = \ln\left(x^2 + 3\right)$		
$f(x) = \sqrt{x^2 + 3}$		

Anti-differentiation by Pattern Recognition

$\frac{d}{dx}[f(g(x))] =$ ______________________________

$\int f'(g(x)) \cdot g'(x)\, dx =$ ______________________________

Find each of the following indefinite integrals by pattern recognition.

$\int 3\cos 3x\,dx$	$\int 2x\sqrt[3]{x^2+5}\,dx$
$\int 2\sin(2x+3)\,dx$	$\int \frac{2x+2}{x^2+2x}\,dx$
$\int \cos(3x+2)dx$	$\int 3x\sqrt{x^2+2}\,dx$
$\int 5e^{3x}\,dx$	$\int \frac{3x}{\sqrt{2x^2+3}}\,dx$

Ant-differentiation by U–Substitution

In each of the eight examples above, the $g'(x)$, or "license to integrate," existed in the integrand of $\int f'(g(x)) \cdot g'(x)\,dx$ or $g'(x)$ was attainable by multiplying by a constant. The $g'(x)$ does not always exist and there are times when it is not attainable by multiplication of a constant. Consider the example below.

$$\int x(2x-1)^3\,dx$$

Identify the "inner function," $g(x)$: ____________________

What is $g'(x)$? ______________ Is $g'(x)$ part of the integrand? ________________

Is $g'(x)$ attainable by multiplying the integrand by a constant? __________________

In this case, we must find the anti-derivative by a method known as U-Substitution. Here is how it works.

1. Let $u =$ the inner function, $g(x)$.	4. Rewrite the entire integrand as a polynomial or polynomial type of function in terms of u. Then, anti-differentiate.
2. Find du and solve the equation for dx.	
3. Find an expression for x in terms of u.	

$$\int \frac{2x+1}{\sqrt{x+4}}\,dx$$

1. Let u = the inner function, $g(x)$.	4. Rewrite the entire integrand as a polynomial or polynomial type of function in terms of u. Then, anti-differentiate.
2. Find du and solve the equation for dx.	
3. Find an expression for x in terms of u.	

Find the value of $\int_0^4 x\sqrt{2x+1}\,dx$. Then, check the result using the graphing calculator.