Interpretations and Applications of the Derivative and the Definite Integral

$$\frac{d}{dx}[\text{AMOUNT}] = \text{The rate at which that amount is changing}$$

For example, if water is being drained from a swimming pool and *R*(*t*) represents the amount of water, measured in cubic feet, that is in a swimming pool at any given time, measured in hours, then $R'(t)$ would represent the rate at which the amount of water is changing.

$$\frac{d}{dx}[R(t)] = R'(t)$$

What would the units of $R'(t)$ be?__________________________

$$\int_a^b \text{RATE} = \text{AMOUNT OF CHANGE}$$

In the context of the example situation above, explain what this value represents: $\int_a^b R'(t)dt = R(b) - R(a)$

The table given below represents the velocity of a particle at given values of *t*, where *t* is measure in minutes.

t **minutes**	0	5	10	15	20	25	30
***v*(*t*)** **ft/minute**	0	1.6	2.7	3.1	2.4	1.6	0

a. Approximate the value of $\int_0^{30} v(t)dt$ using a midpoint Riemann Sum. Using correct units of measure, explain what this value represents.

b. What is the value of $\int_5^{25} a(t)dt$, and using correct units, explain what this value represents.

The temperature of water in a tub at time t is modeled by a strictly increasing, twice differentiable function, W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

Using the data in the table, estimate the value of $W'(12)$. Using correct units, interpret the meaning of this value in the context of this problem.

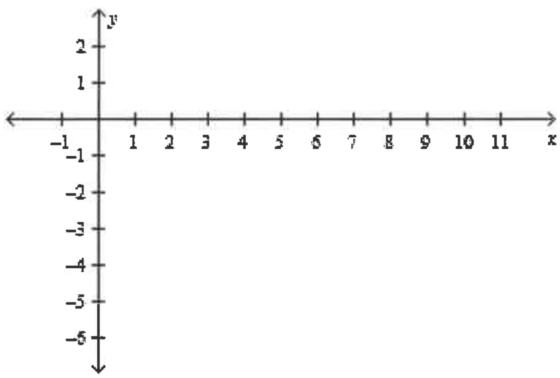
Use the data in the table to evaluate $\int_0^{20} W'(t)dt$. Using correct units, interpret the meaning of this integral in the context of this problem.

For $20 \leq t \leq 25$, the function W that models the water temperature has a first derivative given by the function $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on this model, what is the temperature of the water at time $t = 25$?

A pan of biscuits is removed from an oven at which point in time, $t = 0$ minutes, the temperature of the biscuits is 100°C. The rate at which the temperature of the biscuits is changing is modeled by the function $B'(t) = -13.84e^{-0.173t}$.

Find the value of $B'(3)$. Using correct units, explain the meaning of this value in the context of the problem.

Sketch the graph of $B'(t)$ on the axes below. Explain in the context of the problem why the graph makes sense.



At time $t = 10$, what is the temperature of the biscuits? Show your work.

A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. During the first 5 days of a 60-day period, 3 millimeters of rainfall had been collected. The height of water in the can is modeled by the function, S, where $S(t)$ is measured in millimeters and t is measured in days for $5 \le t \le 60$. The rate at which the height of the water is rising is given by the function $S'(t) = 2\sin(0.03t) + 1.5$.

Find the value of $\int_{10}^{15} S'(t)dt$. Using correct units, explain the meaning of this value in the context of this problem.

At the end of the 60-day period, what is the volume of water that had accumulated in the can? Show your work.

The rate at which people enter an auditorium for a concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$ hours; $R(t)$ is measured in people per hour. V.I.P. tickets were sold to 100 people who are already in the auditorium when the when the doors open at $t = 0$ for general admission ticket holders to enter. The doors close and the concert begins at $t = 2$.

If all of the V.I.P. ticket holders stayed for the start of the concert, how many people are in the auditorium when the concert begins?