

The Fundamental Theorem of Calculus, Particle Motion, and Average Value

Three Things to Always Keep In Mind:

(1) $\int_a^b v(t) dt = p(b) - p(a)$, where $v(t)$ represents the velocity and $p(t)$ represents the position.

(2) $\int_a^b v(t) dt$ = The Net Distance the particle travels on the interval from $t = a$ to $t = b$. If $v(t) > 0$ on the interval (a, b) , then it also represents the Total Distance.

(3) $\int_a^b |v(t)| dt$ = The Total Distance the particle travels on the interval (a, b) , whether or not $v(t) > 0$. To be safe, always do this integral when asked to find total distance when given velocity.

1. The velocity of a particle that is moving along the x – axis is given by the function $v(t) = 3t^2 + 6$. (This is a non-calculator active question.)

a. If the position of the particle at $t = 4$ is 72, what is the position when $t = 2$?

b. What is the total distance the particle travels on the interval $t = 0$ to $t = 7$?

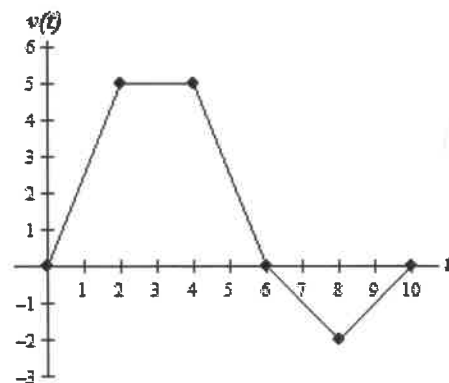
2. The velocity of a particle that is moving along the x – axis is given by the function $v(t) = 0.5e^t(t - 2)^3$.
(This is a calculator active question.)

a. If the position of the particle at $t = 1.5$ is 2.551, what is the position when $t = 3.5$?

b. What is the total distance that the object travels on the interval $t = 1$ to $t = 5$?

The graph of the velocity, measured in feet per second, of a particle moving along the x – axis is pictured below. The position, $p(t)$, of the particle at $t = 8$ is 12. Use the graph of $v(t)$ to answer the questions that follow.

a. What is the position of the particle at $t = 3$?



b. What is the acceleration when $t = 5$?

c. What is the net distance the particle travels from $t = 0$ to $t = 10$?

d. What is the total distance the particle travels from $t = 0$ to $t = 10$?

t	0	3	6	9	12	15	18
$V(t)$	2.3	2.7	2.0	1.3	1.0	1.7	2.1

The table above shows values of the velocity, $V(t)$ in meters per second, of a particle moving along the x -axis at selected values of time, t seconds.

- What does the value of $\int_0^{18} V(t)dt$ represent?
- Using a left Riemann sum of 6 subintervals of equal length, estimate the value of $\int_0^{18} V(t)dt$. Indicate units of measure.
- Using a right Riemann sum of 6 subintervals of equal length, estimate the value of $\int_0^{18} V(t)dt$. Indicate units of measure.
- Using a midpoint Riemann sum of 3 subintervals of equal length, estimate the value of $\int_0^{18} V(t)dt$. Indicate units of measure.
- Using a trapezoidal sum of 6 subintervals of equal length, estimate the value of $\int_0^{18} V(t)dt$. Indicate units of measure.
- Find the average acceleration of the particle from $t = 3$ to $t = 9$. For what value of t , in the table, is this average acceleration approximately equal to $v'(t)$? Explain your reasoning.

