Hello Students,

Here is a packet that contains two weeks' worth of notes and work/practice for the lessons in the near future through April 3rd (excluding Spring Break). Please email me if you have **ANY** questions at:

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Notes/Study Help

9-3 Solving Quadratic Equations

Content Standards

A.REI.4.b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots . . .

Also N.Q.2, A.APR.3, A.CED.1, A.CED.4

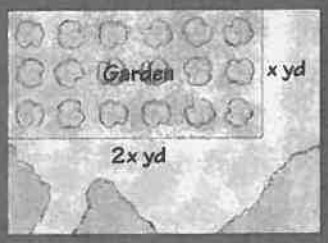
Objective To solve quadratic equations by graphing and using square roots

Make sense of the units. The length, depth, and width need to be the same.

SOLVE IT!

Getting Ready!

The diagram shows a plan for your new garden. You want to use only 1.5 yd^3 of topsoil and plan to spread a layer 4 in. deep. What are the dimensions of the largest garden you can build? How do you know?



MATHEMATICAL PRACTICES The situation in the Solve It can be modeled by a *quadratic equation*.

Lesson Vocabulary
- quadratic equation
- standard form of a quadratic equation
- root of an equation
- zero of a function

take note

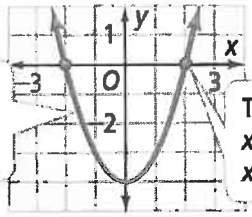
Key Concept Standard Form of a Quadratic Equation

A **quadratic equation** is an equation that can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$. This form is called the **standard form of a quadratic equation.**

Essential Understanding Quadratic equations can be solved by a variety of methods, including graphing and finding square roots.

One way to solve a quadratic equation $ax^2 + bx + c = 0$ is to graph the related quadratic function $y = ax^2 + bx + c$. The solutions of the equation are the x-intercepts of the related function.

To solve $x^2 - 4 = 0$, graph $y = x^2 - 4$.



The solutions of $x^2 - 4 = 0$ are the x-intercepts 2 and −2.

A quadratic equation can have two, one, or no real-number solutions. In a future course you will learn about solutions of quadratic equations that are not real numbers. In this course, *solutions* refers to real-number solutions.

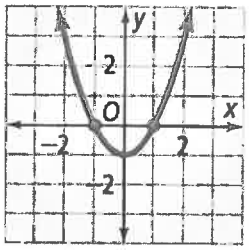
The solutions of a quadratic equation and the x-intercepts of the graph of the related function are often called **roots of the equation** or **zeros of the function.**

Problem 1 Solving by Graphing

What are the solutions of each equation? Use a graph of the related function.

A $x^2 - 1 = 0$

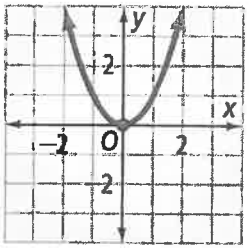
Graph $y = x^2 - 1$.



There are two solutions, ± 1.

B $x^2 = 0$

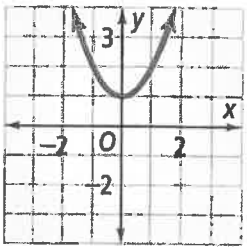
Graph $y = x^2$.



There is one solution, 0.

C $x^2 + 1 = 0$

Graph $y = x^2 + 1$.



There is no real-number solution.

Think

What feature of the graph shows the solutions of the equation?
The x-intercepts show the solutions of the equation.

Got It? **1.** What are the solutions of each equation? Use a graph of the related function.

a. $x^2 - 16 = 0$ **b.** $3x^2 + 6 = 0$ **c.** $x^2 - 25 = -25$

You can solve equations of the form $x^2 = k$ by finding the square roots of each side. For example, the solutions of $x^2 = 81$ are $\pm\sqrt{81}$, or ± 9.

Problem 2 Solving Using Square Roots

What are the solutions of $3x^2 - 75 = 0$?

Think	Write
Write the original equation.	$3x^2 - 75 = 0$
Isolate x^2 on one side of the equation.	$3x^2 = 75$ $x^2 = 25$
Find the square roots of each side and simplify.	$x = \pm\sqrt{25}$ $x = \pm 5$

Plan

How do you know you can solve using square roots?
The equation has an x^2-term and a constant term, but no x-term. So, you can write the equation in the form $x^2 = k$ and then find the square roots of each side.

Got It? **2.** What are the solutions of each equation?

a. $m^2 - 36 = 0$ **b.** $3x^2 + 15 = 0$ **c.** $4d^2 + 16 = 16$

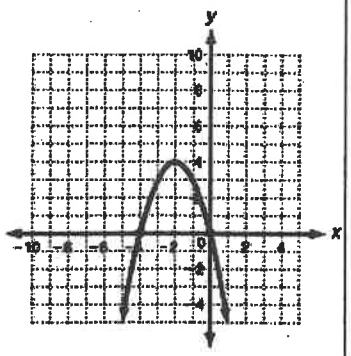
Practice

Algebra 1 8.2 Worksheet
Characteristics of Quadratic Functions

Name _____
Per ___ Date _____

Determine the value of the zeros, the equation of the axis of symmetry, the max or min value and the vertex.

1.



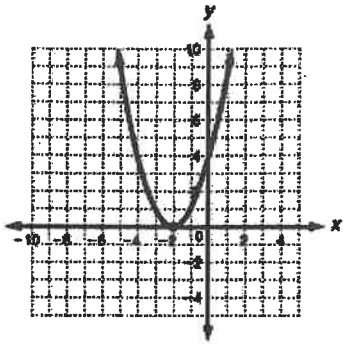
Zeros:

Axis of symmetry:

Max or Min:

Vertex:

2.



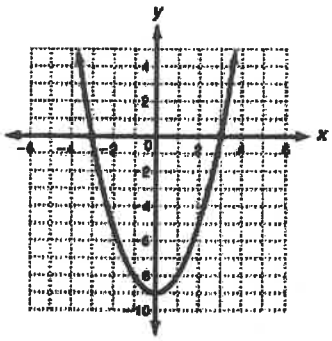
Zeros:

Axis of symmetry:

Max or Min:

Vertex:

3.



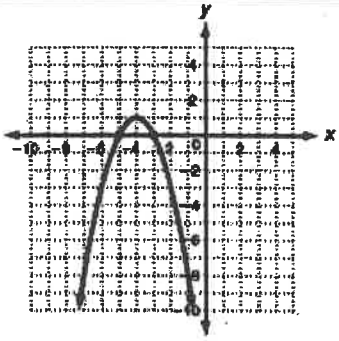
Zeros:

Axis of symmetry:

Max or Min:

Vertex:

4.



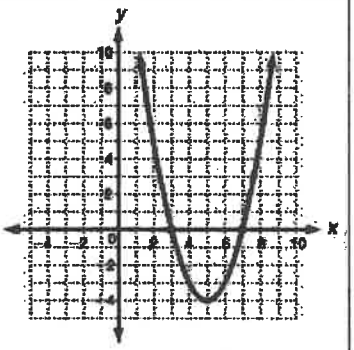
Zeros:

Axis of symmetry:

Max or Min:

Vertex:

5.



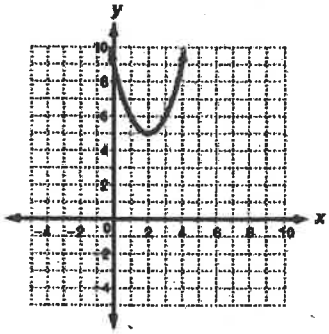
Zeros:

Axis of symmetry:

Max or Min:

Vertex:

6.



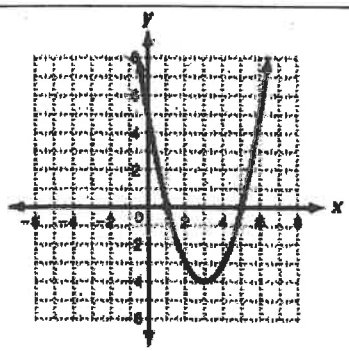
Zeros:

Axis of symmetry:

Max or Min:

Vertex:

7.



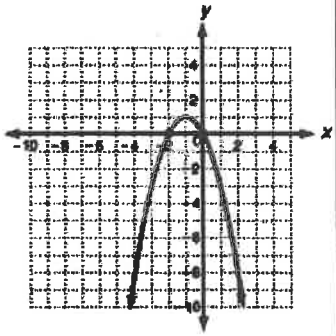
Zeros:

Axis of symmetry:

Max or Min:

Vertex:

8.



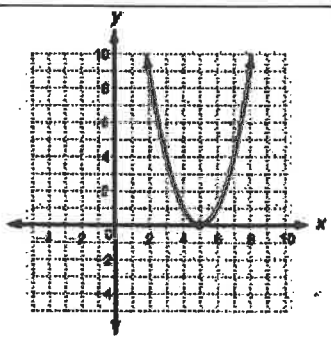
Zeros:

Axis of symmetry:

Max or Min:

Vertex:

9.



Zeros:

Axis of symmetry:

Max or Min:

Vertex:

Graphing Quadratics Review Worksheet

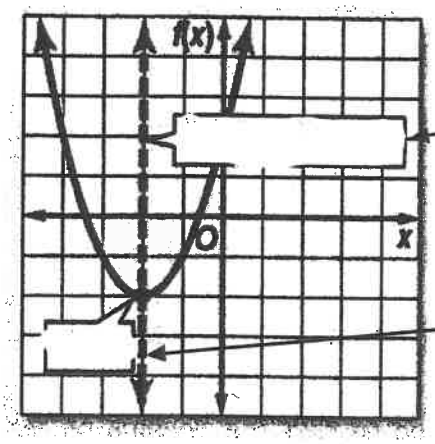
Name _____

Fill in each blank using the word bank.

vertex	minimum	axis of symmetry	x-intercepts
parabola	maximum	zeros/roots	$ax^2 + bx + c$

1. Standard form of a quadratic function is y = _____

2. The shape of a quadratic equation is called a _____



3. _____

4. _____

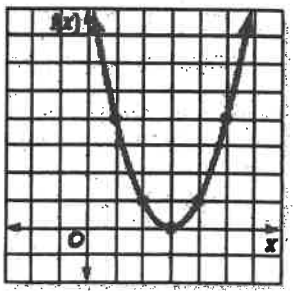
5. When the vertex is the highest point on the graph, we call that a _____.

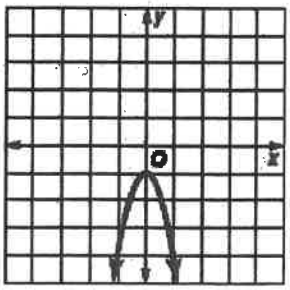
6. When the vertex is the lowest point on the graph, we call that a _____.

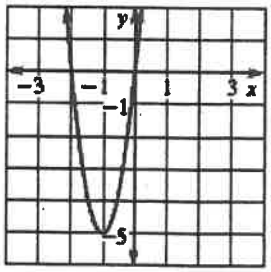
7. Our solutions are the _____.

8. Solutions to quadratic equations are called _____.

Determine whether the quadratic functions have two real roots, one real root, or no real roots. If possible, list the zeros of the function.







9. Number of roots: _____

Zero(s): _____

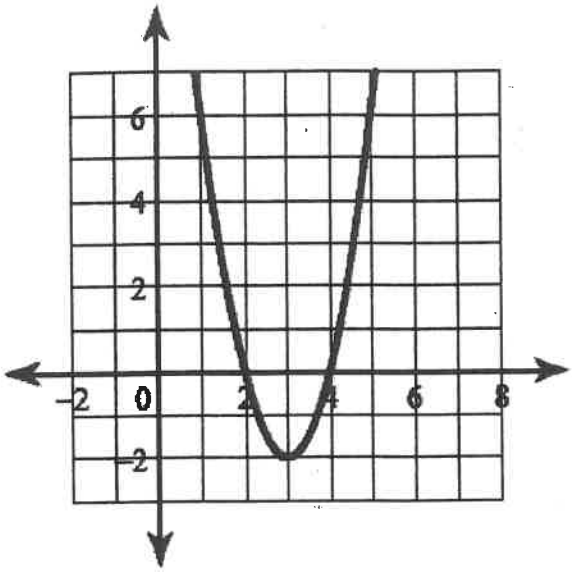
10. Number of roots: _____

Zero(s): _____

11. Number of roots: _____

Zero(s): _____

12. Given the graph, identify the following.



Axis of symmetry: _____

Vertex: _____

How many zeros: _____ which are: _____

Domain: _____

Range: _____

Graph the following quadratic functions by using critical values and/or factoring.

You need three points to graph and don't necessarily need all the information listed.

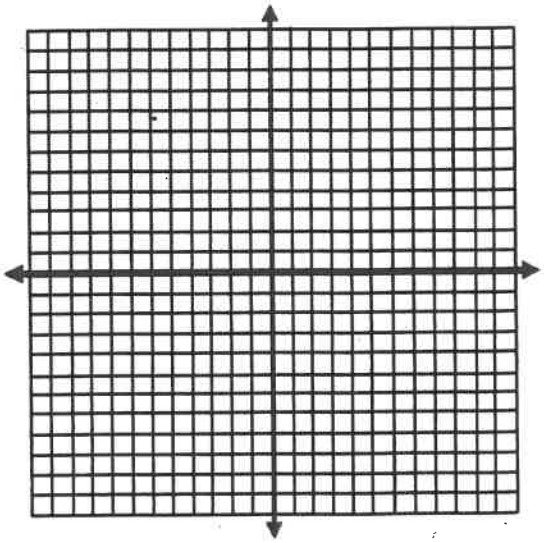
Remember: Option 1: If it factors, find the zeros.

The middle of the two factors is the axis of symmetry.

Option 2: If it doesn't factor, find the axis of symmetry with $x = \frac{-b}{2a}$

Plug the x-value into the original equation to find the y-value of the vertex. The y-intercept is at (0, c)

13. $y = x^2 - 2x - 3$ factor or critical values?



Identify the zeros/roots: _____ and _____

Does it have a minimum or maximum? _____

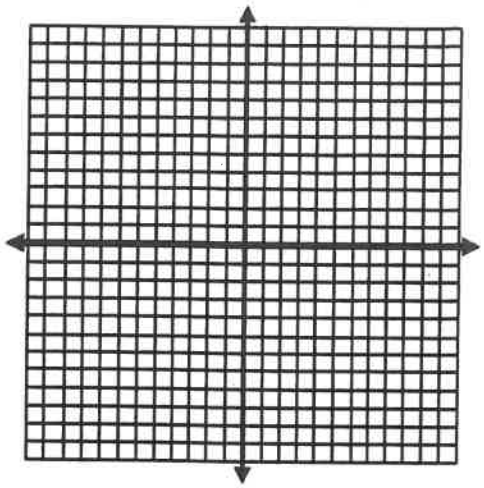
Axis of symmetry: _____ Vertex: _____

y-intercept: _____

Domain: _____ Range: _____

Graph at least 5 points

14. $y = -x^2 - 4x + 5$ factor or critical values?



Identify the zeros/roots: _____ and _____

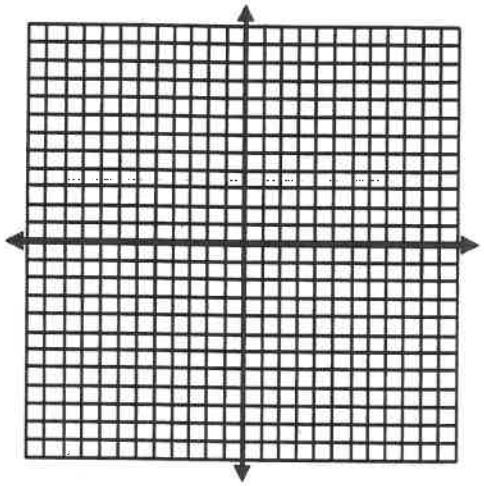
Does it have a minimum or maximum? _____

Axis of symmetry: _____ Vertex: _____

y-intercept: _____ Graph at least 5 points

Domain: _____ Range: _____

15. $y = x^2 + 4x + 7$ factor or critical values?

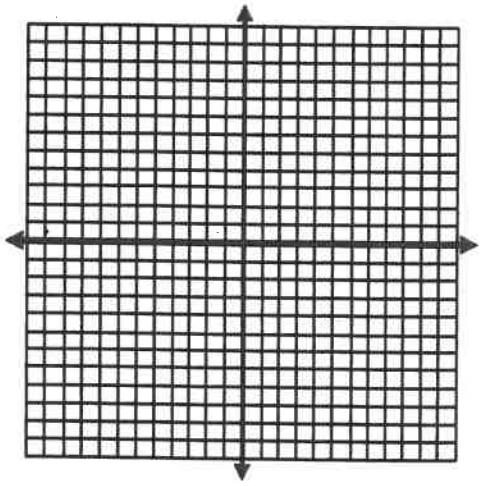


Axis of symmetry: _____ Vertex: _____

Max or Min? _____

y-intercept: _____ Graph at least 3 points

16. $y = -x^2 - 2x + 2$ factor or critical values?



Axis of symmetry: _____ Vertex: _____

Max or Min? _____

y-intercept: _____ Graph at least 5 points

17. A bottlenose dolphin jumps out of the water. The path the dolphin travels can be modeled by $h = -0.2d^2 + 2d$, where h represents the height of the dolphin and d represents horizontal distance.

a. What is the maximum height the dolphin reaches?

b. How far did the dolphin jump?

Notes/Study Help

9-6 The Quadratic Formula and the Discriminant

Content Standards

A.REI.4.a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ . . . Derive the quadratic formula from this form.

Also N.Q.3, A.CED.1, A.REI.4.b

Objectives To solve quadratic equations using the quadratic formula
To find the number of solutions of a quadratic equation

You have several ways to solve this problem: graphing, making a table, and factoring. Which will you use?

SOLVE IT!

Getting Ready!

Your friend's aunt has a brick walkway in her backyard. Her plan is to decrease the length by the same amount she increases the width to make a rectangular patio. She wants the patio to have an area of 310 ft^2. Can she build a patio to meet her plan? Explain your reasoning.

MATHEMATICAL PRACTICES Recall that quadratic equations can have two, one, or no real-number solutions. A quadratic equation can never have more than two solutions.

Lesson Vocabulary
- quadratic formula
- discriminant

Essential Understanding You can find the solution(s) of *any* quadratic equation using the **quadratic formula**.

take note

Key Concept Quadratic Formula

Algebra

If $ax^2 + bx + c = 0$, and $a \neq 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example

Suppose $2x^2 + 3x - 5 = 0$. Then $a = 2$, $b = 3$, and $c = -5$. Therefore

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-5)}}{2(2)}$$

Here's Why It Works If you complete the square for the general equation $ax^2 + bx + c = 0$, you can derive the quadratic formula.

Step 1 Write $ax^2 + bx + c = 0$ so the coefficient of x^2 is 1.

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad \text{Divide each side by } a.$$

Step 2 Complete the square.

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \quad \text{Subtract } \frac{c}{a} \text{ from each side.}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 \quad \text{Add } \left(\frac{b}{2a}\right)^2 \text{ to each side.}$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2} \quad \text{Write the left side as a square.}$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2} \quad \text{Multiply } -\frac{c}{a} \text{ by } \frac{4a}{4a} \text{ to get like denominators.}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \quad \text{Simplify the right side.}$$

Step 3 Solve the equation for x.

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \quad \text{Take square roots of each side.}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} \quad \text{Simplify the right side.}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \quad \text{Subtract } \frac{b}{2a} \text{ from each side.}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Simplify.}$$

This step uses the property $\sqrt{\frac{m}{n}} = \frac{\sqrt{m}}{\sqrt{n}}$, which you will study in Lesson 10-2.

Be sure to write a quadratic equation in standard form before using the quadratic formula.

Problem 1 Using the Quadratic Formula

What are the solutions of $x^2 - 8 = 2x$? Use the quadratic formula.

$$x^2 - 2x - 8 = 0 \quad \text{Write the equation in standard form.}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Use the quadratic formula.}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2(1)} \quad \text{Substitute 1 for } a, -2 \text{ for } b, \text{ and } -8 \text{ for } c.$$

$$x = \frac{2 \pm \sqrt{36}}{2} \quad \text{Simplify.}$$

$$x = \frac{2 + 6}{2} \quad \text{or} \quad x = \frac{2 - 6}{2} \quad \text{Write as two equations.}$$

$$x = 4 \quad \text{or} \quad x = -2 \quad \text{Simplify.}$$

Think

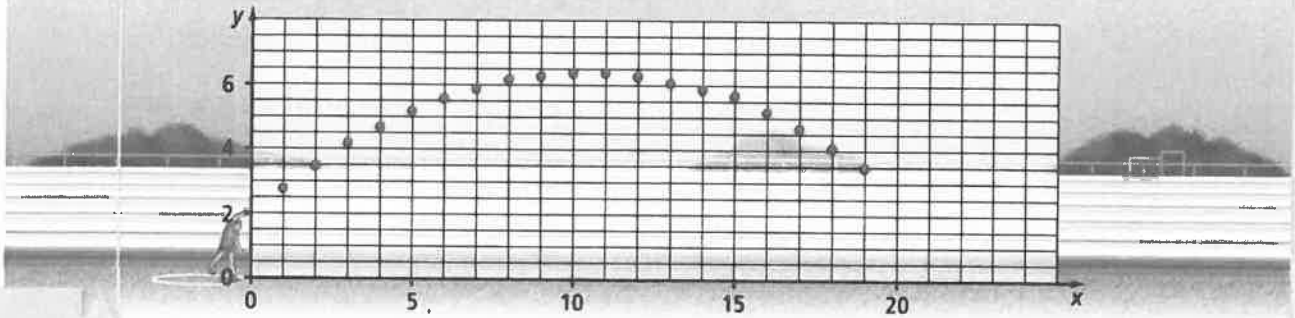
Why do you need to write the equation in standard form?
You can only use the quadratic formula with equations in the form $ax^2 + bx + c = 0$.

Got It? 1. What are the solutions of $x^2 - 4x = 21$? Use the quadratic formula.

When the radicand in the quadratic formula is not a perfect square, you can use a calculator to approximate the solutions of an equation.

Problem 2 Finding Approximate Solutions

Sports In the shot put, an athlete throws a heavy metal ball through the air. The arc of the ball can be modeled by the equation $y = -0.04x^2 + 0.84x + 2$, where x is the horizontal distance, in meters, from the athlete and y is the height, in meters, of the ball. How far from the athlete will the ball land?



Think

Why do you substitute 0 for y? When the ball hits the ground, its height will be 0.

$0 = -0.04x^2 + 0.84x + 2$			Substitute 0 for y in the given equation.
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$			Use the quadratic formula.
$x = \dfrac{-0.84 \pm \sqrt{0.84^2 - 4(-0.04)(2)}}{2(-0.04)}$			Substitute -0.04 for a, 0.84 for b, and 2 for c.
$x = \dfrac{-0.84 \pm \sqrt{1.0256}}{-0.08}$			Simplify.
$x = \dfrac{-0.84 + \sqrt{1.0256}}{-0.08}$	or	$x = \dfrac{-0.84 - \sqrt{1.0256}}{-0.08}$	Write as two equations.
$x \approx -2.16$	or	$x \approx 23.16$	Simplify.

Only the positive answer makes sense in this situation. The ball will land about 23.16 m from the athlete.

Got It? 2. A batter strikes a baseball. The equation $y = -0.005x^2 + 0.7x + 3.5$ models its path, where x is the horizontal distance, in feet, the ball travels and y is the height, in feet, of the ball. How far from the batter will the ball land? Round to the nearest tenth of a foot.

There are many methods for solving a quadratic equation.

Method	When to Use
Graphing	Use if you have a graphing calculator handy.
Square roots	Use if the equation has no x-term.
Factoring	Use if you can factor the equation easily.
Completing the square	Use if the coefficient of x^2 is 1, but you cannot easily factor the equation.
Quadratic formula	Use if the equation cannot be factored easily or at all.

Problem 3 Choosing an Appropriate Method

Think

Can you use the quadratic formula to solve part (A)?
Yes. You can use the quadratic formula with $a = 3$, $b = 0$, and $c = -9$. However, it is faster to use square roots.

Which method(s) would you choose to solve each equation? Explain your reasoning.

A $3x^2 - 9 = 0$	Square roots; there is no x-term
B $x^2 - x - 30 = 0$	Factoring; the equation is easily factorable
C $6x^2 + 13x - 17 = 0$	Quadratic formula, graphing; the equation cannot be factored
D $x^2 - 5x + 3 = 0$	Quadratic formula, completing the square, or graphing; the coefficient of the x^2-term is 1, but the equation cannot be factored
E $-16x^2 - 50x + 21 = 0$	Quadratic formula, graphing; the equation cannot be factored easily since the numbers are large

Got It? 3. Which method(s) would you choose to solve each equation? Justify your reasoning.

a. $x^2 - 8x + 12 = 0$ **b.** $169x^2 = 36$ **c.** $5x^2 + 13x - 1 = 0$

Quadratic equations can have two, one, or no real-number solutions. Before you solve a quadratic equation, you can determine how many real-number solutions it has by using the discriminant. The **discriminant** is the expression under the radical sign in the quadratic formula.

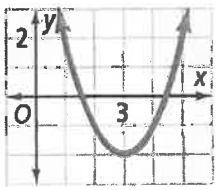
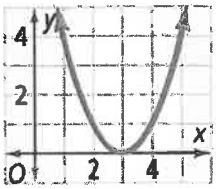
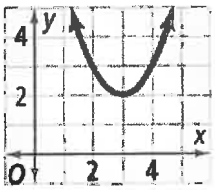
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

the discriminant

The discriminant of a quadratic equation can be positive, zero, or negative.

take note

Key Concept Using the Discriminant

Discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Example	$x^2 - 6x + 7 = 0$ The discriminant is $(-6)^2 - 4(1)(7) = 8$, which is positive. 	$x^2 - 6x + 9 = 0$ The discriminant is $(-6)^2 - 4(1)(9) = 0$. 	$x^2 - 6x + 11 = 0$ The discriminant is $(-6)^2 - 4(1)(11) = -8$, which is negative. 
Number of Solutions	There are two real-number solutions.	There is one real-number solution.	There are no real-number solutions.

Problem 4 Using the Discriminant

How many real-number solutions does $2x^2 - 3x = -5$ have?

Plan

Can you solve this problem another way?
Yes. You could actually solve the equation to find any solutions. However, you only need to know the number of solutions, so use the discriminant.

Think	Write
Write the equation in standard form.	$2x^2 - 3x + 5 = 0$
Evaluate the discriminant by substituting 2 for *a*, −3 for *b*, and 5 for *c*.	$b^2 - 4ac = (-3)^2 - 4(2)(5)$ $= -31$
Draw a conclusion.	Because the discriminant is negative, the equation has no real-number solutions.

Got It? **4. a.** How many real-number solutions does $6x^2 - 5x = 7$ have?
b. Reasoning If a is positive and c is negative, how many real-number solutions will the equation $ax^2 + bx + c = 0$ have? Explain.

Lesson Check

Do you know HOW?

Use the quadratic formula to solve each equation. If necessary, round answers to the nearest hundredth.

1. $-3x^2 - 11x + 4 = 0$
2. $7x^2 - 2x = 8$
3. How many real-number solutions does the equation $-2x^2 + 8x - 5 = 0$ have?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. **Vocabulary** Explain how the discriminant of the equation $ax^2 + bx + c = 0$ is related to the number of x-intercepts of the graph of $y = ax^2 + bx + c$.
5. **Reasoning** What method would you use to solve the equation $x^2 + 9x + c = 0$ if $c = 14$? If $c = 7$? Explain.
6. **Writing** Explain how completing the square is used to derive the quadratic formula.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Use the quadratic formula to solve each equation. See Problem 1.

7. $2x^2 + 5x + 3 = 0$
8. $5x^2 + 16x - 84 = 0$
9. $4x^2 + 7x - 15 = 0$
10. $3x^2 - 41x = -110$
11. $18x^2 - 45x - 50 = 0$
12. $3x^2 + 44x = -96$
13. $3x^2 + 19x = 154$
14. $2x^2 - x - 120 = 0$
15. $5x^2 - 47x = 156$

Practice

Name_____

Understanding the Discriminant

Date_____ Period____

Find the value of the discriminant of each quadratic equation.

1) $6p^2 - 2p - 3 = 0$

2) $-2x^2 - x - 1 = 0$

3) $-4m^2 - 4m + 5 = 0$

4) $5b^2 + b - 2 = 0$

5) $r^2 + 5r + 2 = 0$

6) $2p^2 + 5p - 4 = 0$

Find the discriminant of each quadratic equation then state the numberof real and imaginary solutions.

7) $9n^2 - 3n - 8 = -10$

8) $-2x^2 - 8x - 14 = -6$

9) $9m^2 + 6m + 6 = 5$

10) $4a^2 = 8a - 4$

11) $-9b^2 = -8b + 8$

12) $-x^2 - 9 = 6x$

13) $-4r^2 - 4r = 6$

14) $8b^2 - 6b + 3 = 5b^2$

Find the discriminant then state the number of rational, irrational, and imaginary solutions.

15) $-6x^2 - 6 = -7x - 9$

16) $4k^2 + 5k + 4 = -3k$

17) $-7n^2 + 16n = 8n$

18) $2x^2 = 10x + 5$

19) $-10n^2 - 3n - 9 = -2n$

20) $-9r^2 - 8r - 1 = r - r^2 - 9$

21) $-3p^2 + 10p + 5 = -8p^2$

22) $m^2 + 5m = 2m^2$

Critical thinking questions:

23) Write a quadratic equation that has two imaginary solutions.

24) In your own words explain why a quadratic equation can't have one imaginary solution.

Notes/Study Help

9-4 Factoring to Solve Quadratic Equations

Content Standards

A.REI.4.b Solve quadratic equations by . . . factoring . . .

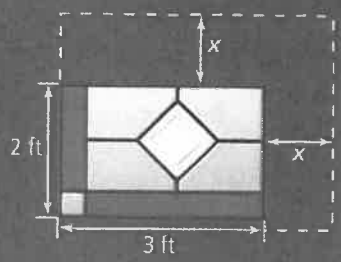
Also A.SSE.3.a, A.CED.1, F.IF.8.a

Objective To solve quadratic equations by factoring

Analyze the situation. What expressions can you use for the length and for the width?

SOLVE IT! **Getting Ready!**

You are finishing a stained glass hanging that your friend has started. You have enough supplies to add 6 ft^2 to the hanging. You are planning to add the same amount to the length and width. What will be the dimensions of the hanging when you are finished? How do you know?



MATHEMATICAL PRACTICES In the previous lesson, you solved quadratic equations $ax^2 + bx + c = 0$ by finding square roots. This method works if $b = 0$.

Lesson Vocabulary
- Zero-Product Property

Essential Understanding You can solve some quadratic equations, including equations where $b \neq 0$, by using the *Zero-Product Property*.

The Multiplication Property of Zero states that for any real number a, $a \cdot 0 = 0$. This is equivalent to the following statement: For any real numbers a and b, if $a = 0$ or $b = 0$, then $ab = 0$. The Zero-Product Property reverses this statement.

take note

Property Zero-Product Property

For any real numbers a and b, if $ab = 0$, then $a = 0$ or $b = 0$.

Example If $(x + 3)(x + 2) = 0$, then $x + 3 = 0$ or $x + 2 = 0$.

Problem 1 Using the Zero-Product Property

What are the solutions of the equation $(4t + 1)(t - 2) = 0$?

$(4t + 1)(t - 2) = 0$

$4t + 1 = 0$ or $t - 2 = 0$ Use the Zero-Product Property.

$4t = -1$ or $t = 2$ Solve for t.

$t = -\frac{1}{4}$ or $t = 2$

Think

How else can you write the solutions? You can write the solutions as a set in roster form: $\{-\frac{1}{4}, 2\}$.

Got It? **1.** What are the solutions of each equation?

a. $(x + 1)(x - 5) = 0$ **b.** $(2x + 3)(x - 4) = 0$

c. $(2y + 1)(y + 14) = 0$ **d.** $(7n - 2)(5n - 4) = 0$

You can also use the Zero-Product Property to solve equations of the form $ax^2 + bx + c = 0$ if the quadratic expression $ax^2 + bx + c$ can be factored.

Problem 2 Solving by Factoring

Multiple Choice **What are the solutions of the equation $x^2 + 8x + 15 = 0$?**

Ⓐ $-5, -3$ Ⓒ $-3, 5$

Ⓑ $-5, 3$ Ⓓ $3, 5$

Plan

How can you factor $x^2 + 8x + 15$?
Find two integers with a product of 15 and a sum of 8.

$x^2 + 8x + 15 = 0$

$(x + 3)(x + 5) = 0$ Factor $x^2 + 8x + 15$.

$x + 3 = 0$ or $x + 5 = 0$ Use the Zero-Product Property.

$x = -3$ or $x = -5$ Solve for x.

The solutions are -3 and -5. The correct answer is A.

Got It? **2.** What are the solutions of each equation?

a. $m^2 - 5m - 14 = 0$ **b.** $p^2 + p - 20 = 0$ **c.** $2a^2 - 15a + 18 = 0$

Before solving a quadratic equation, you may need to add or subtract terms from each side in order to write the equation in standard form. Then factor the quadratic expression.

Problem 3 Writing in Standard Form First

What are the solutions of $4x^2 - 21x = 18$?

Think

Why do you need to subtract 18 from each side before you factor?
To use the Zero-Product Property, one side of the equation must be zero.

$4x^2 - 21x = 18$

$4x^2 - 21x - 18 = 0$ Subtract 18 from each side.

$(4x + 3)(x - 6) = 0$ Factor $4x^2 - 21x - 18$.

$4x + 3 = 0$ or $x - 6 = 0$ Use the Zero-Product Property.

$4x = -3$ or $x = 6$ Solve for x.

$x = -\frac{3}{4}$ or $x = 6$

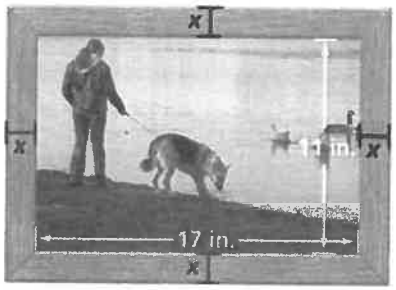
The solutions are $-\frac{3}{4}$ and 6.

Got It? **3. a.** What are the solutions of $x^2 + 14x = -49$?

b. Reasoning Why do quadratic equations of the form $x^2 + 2ax + a^2 = 0$ or $x^2 - 2ax + a^2 = 0$ have only one real-number solution?

Problem 4 Using Factoring to Solve a Real-World Problem

Photography You are constructing a frame for the rectangular photo shown. You want the frame to be the same width all the way around and the total area of the frame and photo to be 315 in.^{2}. What should the outer dimensions of the frame be?



Know
The size of the photo is 11 in. by 17 in. The total area is 315 in.^{2}.

Need
The outer dimensions of the frame

Plan
Write the frame's outer dimensions in terms of its width x. Use these dimensions to write an equation for the area of the frame and photo.

$(2x + 11)(2x + 17) = 315$	Width × Length = Area
$4x^2 + 56x + 187 = 315$	Find the product $(2x + 11)(2x + 17)$.
$4x^2 + 56x - 128 = 0$	Subtract 315 from each side.
$4(x^2 + 14x - 32) = 0$	Factor out 4.
$4(x + 16)(x - 2) = 0$	Factor $x^2 + 14x - 32$.
$x + 16 = 0$ or $x - 2 = 0$	Use the Zero-Product Property.
$x = -16$ or $x = 2$	Solve for x.

The only reasonable solution is 2. So the outer dimensions of the frame are $2(2) + 11$ in. by $2(2) + 17$ in., or 15 in. by 21 in.

Think
Why can you ignore the factor of 4?
By the Zero-Product Property, one of the factors, 4, $x + 16$, or $x - 2$, must equal 0. Since $4 \neq 0$, either $x + 16$ or $x - 2$ equals 0.

Got It? **4.** In Problem 4, suppose the total area of the frame and photo were 391 in.^{2}. What would the outer dimensions of the frame be?

Lesson Check

Do you know HOW?

Solve each equation.

1. $(v - 4)(v - 7) = 0$
2. $t^2 + 3t - 54 = 0$
3. $3y^2 - 17y + 24 = 0$

STEM **4. Carpentry** You are making a rectangular table. The area of the table should be 10 ft^2. You want the length of the table to be 1 ft shorter than twice its width. What should the dimensions of the table be?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Vocabulary Give an example of how the Zero-Product Property can be used to solve a quadratic equation.

6. Compare and Contrast How is factoring the expression $x^2 - 6x + 8$ similar to solving the equation $x^2 - 6x + 8 = 0$? How is it different?

7. Reasoning Can you extend the Zero-Product Property to nonzero products of numbers? For example, if $ab = 8$, is it always true that $a = 8$ or $b = 8$? Explain.

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Use the Zero-Product Property to solve each equation. See Problem 1.

8. $(x - 9)(x - 8) = 0$ **9.** $(4k + 5)(k + 7) = 0$ **10.** $n(n + 2) = 0$

11. $-3n(2n - 5) = 0$ **12.** $(7x + 2)(5x - 4) = 0$ **13.** $(4a - 7)(3a + 8) = 0$

Solve by factoring. See Problems 2 and 3.

14. $x^2 + 11x + 10 = 0$ **15.** $g^2 + 4g - 32 = 0$ **16.** $s^2 - 14s + 45 = 0$

17. $2z^2 - 21z - 36 = 0$ **18.** $3q^2 + q - 14 = 0$ **19.** $4m^2 - 27m - 40 = 0$

20. $x^2 + 13x = -42$ **21.** $p^2 - 4p = 21$ **22.** $c^2 = 5c$

23. $2w^2 - 11w = -12$ **24.** $3h^2 + 17h = -10$ **25.** $9b^2 = 16$

26. Geometry A box shaped like a rectangular prism has a volume of 280 in.^{3}. Its dimensions are 4 in. by $(n + 2)$ in. by $(n + 5)$ in. Find n. See Problem 4.

27. Knitting You are knitting a blanket. You want the area of the blanket to be 24 ft^2. You want the length of the blanket to be 2 ft longer than its width. What should the dimensions of the blanket be?

STEM **28. Construction** You are building a rectangular deck. The area of the deck should be 250 ft^2. You want the length of the deck to be 5 ft longer than twice its width. What should the dimensions of the deck be?

B Apply

Use the Zero-Product Property to solve each equation. Write your solutions as a set in roster form.

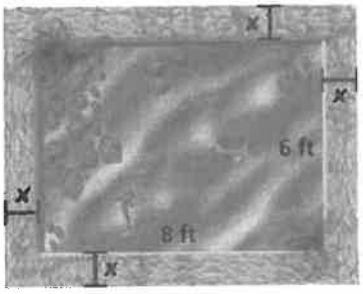
29. $x^2 + 6x + 8 = 0$ **30.** $a^2 + 8a + 12 = 0$ **31.** $k^2 + 7k + 10 = 0$

Write each equation in standard form. Then solve.

32. $7n^2 + 16n + 15 = 2n^2 + 3$ **33.** $4q^2 + 3q = 3q^2 - 4q + 18$

34. Think About a Plan You have a rectangular koi pond that measures 6 ft by 8 ft. You have enough concrete to cover 72 ft^2 for a walkway, as shown in the diagram. What should the width of the walkway be?

- How can you write the outer dimensions of the walkway?
- How can you represent the total area of the walkway and pond in two ways?



35. Reasoning Find the zeros of the function $f(x) = x^2 - 3x + 2$ by factoring. How can you verify the zeros of the function are correct by looking at the graph?

36. Error Analysis Describe and correct the error made in solving the equation.

$$\cancel{\begin{aligned} 2x^2 + 3x &= 20 \\ x(2x + 3) &= 20 \\ x = 0 \text{ or } 2x + 3 &= 0 \\ x = 0 \text{ or } x &= -\tfrac{3}{2} \end{aligned}}$$

37. Reasoning How many solutions does an equation of the form $x^2 - k^2 = 0$ have? Explain.

38. Sports You throw a softball into the air with an initial upward velocity of 38 ft/s and an initial height of 5 ft.

a. Use the vertical motion model to write an equation that gives the ball's height h, in feet, at time t, in seconds.

b. The ball's height is 0 ft when it is on the ground. Solve the equation you wrote in part (a) for $h = 0$ to find when the ball lands.

Solve each cubic equation by factoring out the GCF first.

39. $x^3 - 10x^2 + 24x = 0$ **40.** $x^3 - 5x^2 + 4x = 0$ **41.** $3x^3 - 9x^2 = 0$

Challenge

42. Find an equation that has the given numbers as solutions. For example, 4 and −3 are solutions of $x^2 - x - 12 = 0$.

a. −5, 8 **b.** 3, −2 **c.** $\frac{1}{2}$, −10 **d.** $\frac{2}{3}$, $-\frac{5}{7}$

Solve. Factor by grouping.

43. $x^3 + 5x^2 - x - 5 = 0$ **44.** $x^3 + x^2 - 4x - 4 = 0$ **45.** $x^3 + 2x^2 - 9x - 18 = 0$

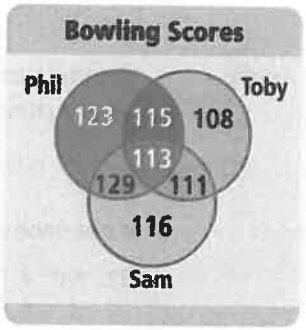
Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

46. What is the negative solution of the equation $2x^2 - 13x - 7 = 0$? Round to the nearest thousandth.

47. Phil, Toby, and Sam bowled four games last weekend. Their scores are shown in the Venn diagram at the right. What is the highest score that only Toby and Sam have in common?



48. What is the y-intercept of the line with equation $3y - 4x = 9$?

49. How many elements are in the union of the two sets $M = \{1, 2, -3, 4\}$ and $N = \{1, -2, 3, 5\}$?

50. A rectangular card has a length 1 in. longer than twice the width and an area of 15 in.^{2}. What is the width of the card, in inches?

Mixed Review

Solve each equation by finding square roots. If the equation has no real-number solution, write *no solution*. See Lesson 9-3.

51. $t^2 = 144$ **52.** $w^2 - 8 = -17$ **53.** $b^2 + 100 = 100$

54. $5h^2 - 80 = 0$ **55.** $49 - m^2 = 0$ **56.** $3q^2 = 27$

Get Ready! **To prepare for Lesson 9-5, do Exercises 57–59.**

Factor each expression. See Lesson 8-7.

57. $y^2 - 10y + 25$ **58.** $g^2 - 14g + 49$ **59.** $m^2 + 18m + 81$

Practice

Algebra 2 (Conway) Name_____

5.2.5 - Solving Quadratic Equations by Factoring

Date_____ Period____

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Solve each equation by factoring.

1) $b^2 + b - 30 = 0$

2) $x^2 + 12x + 27 = 0$

3) $n^2 + 6n - 7 = 0$

4) $n^2 - 2n - 99 = 0$

5) $a^2 - 144 = 0$

6) $p^2 + 13p + 30 = 0$

7) $x^2 - 12x + 20 = 0$

8) $n^2 - 121 = 0$

9) $m^2 + 18m + 72 = 0$

10) $8r^2 - 40r - 112 = 0$

11) $11x^2 - 176 = 0$

12) $b^2 + b - 30 = 0$

13) $n^2 - 100 = 0$

14) $7v^2 - 42v - 49 = 0$

Elementary Algebra Skill

Solving Quadratic Equations by Factoring

Solve each equation by factoring.

1) $x^2 - 9x + 18 = 0$

2) $x^2 + 5x + 4 = 0$

3) $n^2 - 64 = 0$

4) $b^2 + 5b = 0$

5) $35n^2 + 22n + 3 = 0$

6) $15b^2 + 4b - 4 = 0$

7) $7p^2 - 38p - 24 = 0$

8) $3x^2 + 14x - 49 = 0$

9) $3k^2 - 18k - 21 = 0$

10) $6k^2 - 42k + 72 = 0$

11) $x^2 = 11x - 28$

12) $k^2 + 15k = -56$

13) $3m^2 = -16m - 21$

14) $8x^2 = 30 + 43x$

15) $x^2 + 17x + 49 = 3x$

16) $m^2 = 2m$

17) $2k^2 - 14 = -3k$

18) $3v^2 + 36v + 49 = 8v$

19) $10x^2 - 26x = -12$

20) $15p^2 + 80 = -80p$

Name:_____

Spring Break AIR Test Review

This work is due 4/1 during class. Please complete the packet completely showing work when needed. To receive EXTRA CREDIT for the 4th quarter, take a selfie with your packet and email it to me by 4/1 at 9:00am or bring a printed copy to class. You will NOT receive extra credit for pictures on your phone.

1. A total of 330 children and adults attended a school play. There were 21 times as many children in attendance as there were adults.

   This situation is modeled by the given system of equations, where a represents the number of adults and c represents the number of children.

   $c = 21a$
   $a + c = 330$

   How many children attended the play?

2. For the question, I would graph the points on DESMOS and then try to make a determination (a scatter plot).

   A new car wash business records the number of cars it washes each day for the first 7 days the business is open. The data are shown in the table.

Day	Number of Cars Washed
1	41
2	44
3	48
4	52
5	57
6	56
7	57

   The car wash owner uses the equation $y = x + 44$ to model the data, where x represents the number of days the business has been open and y represents the number of cars washed.

   Which explanation **best** describes whether the equation is a good fit for the data?

   Ⓐ The equation is a good fit because the residual points are approximately linear.

   Ⓑ The equation is a good fit because the residual points have a positive association.

   Ⓒ The equation is not a good fit because a residual plot with the pattern from this data set indicates a bad fit.

   Ⓓ The equation is not a good fit because there should be an equal number of points below and above the x-axis in the residual plot.

3. This item has three parts.

Eleanor incorrectly solves the equation $\frac{1}{2}(x + 18) = 4(2x - 6) - 9x$.

Part A. Select the first equation in which Eleanor makes an error.

Step	Equation
Given	$\frac{1}{2}(x+18)=4(2x-6)-9x$
1.	$x + 18 = 8(2x - 6) - 9x$
2.	$x + 18 = 16x - 48 - 9x$
3.	$x + 18 = 7x - 48$
4.	$66 = 6x$
5.	$x = 11$

Part B. Create an equation to correct Eleanor's error identified in Part A.

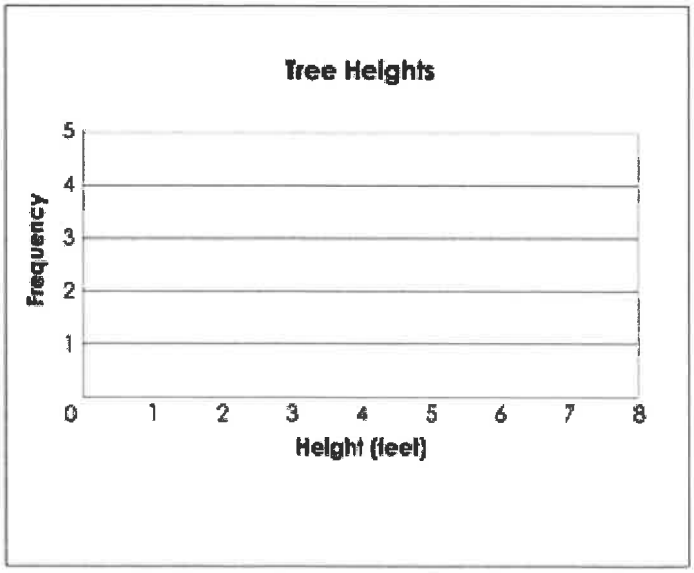
Part C. What is the correct solution to $\frac{1}{2}(x + 18) = 4(2x - 6) - 9x$?

$x =$

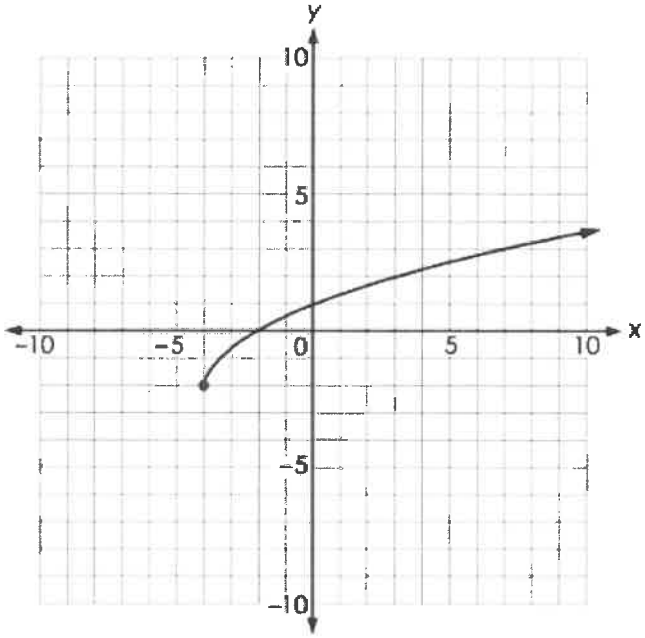
4. A landscaper records the heights, in feet, of 15 newly planted trees in a community garden, as shown.

3.2,	4.3,	3.5,	5.4,	3.7,
5.5,	6.2,	3.1,	6.8,	7.1,
4.8,	6.5,	4.9,	5.3,	5.9

Complete the histogram by selecting frequencies for the heights of the newly planted trees in the community garden.



5. The graph of a function is shown.



What is the domain of the function?

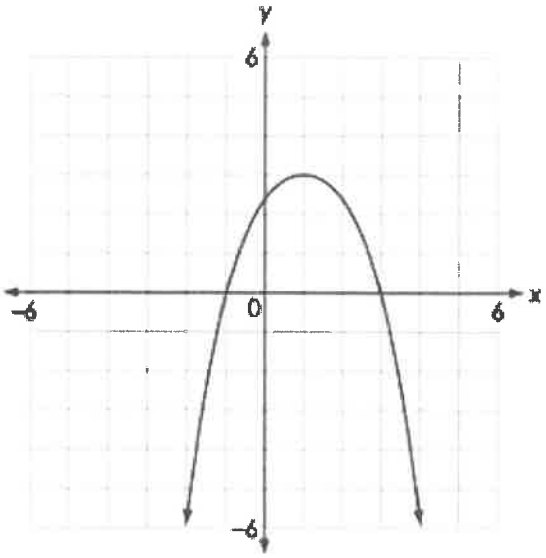
Ⓐ $x \geq -4$

Ⓑ $x \geq -2$

Ⓒ $x \geq 0$

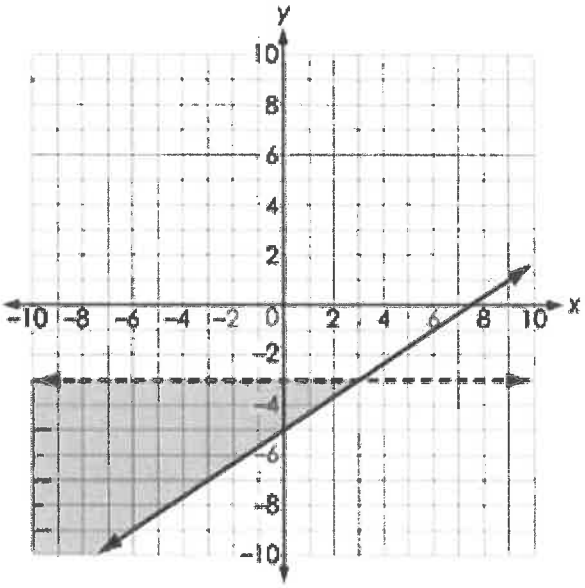
Ⓓ $x \geq 1$

6. The graph of a function is shown.



What is the maximum value of the function?

7. The graph of a system of inequalities is shown.



Create the system of inequalities that is represented by the graph.

8. A function is given.

$f(x) = 2^x + 3$

What is the value of $f(-2)$?

$f(-2) =$ _____

9. A factory has two assembly lines, M and N, that make the same toy. On Monday, only assembly line M was functioning, and it made 900 toys.

On Tuesday, both assembly lines were functioning for the same amount of time. Line M made 300 toys per hour and line N made 480 toys per hour. Line N made as many toys on Tuesday as line M did over both days.

Write an equation that can be used to find the number of hours, t, that the assembly lines were functioning on Tuesday.

10. Select the most appropriate unit for each situation.

	$\frac{\text{feet}}{\text{minute}}$	$\frac{\text{square feet}}{\text{minute}}$	$\frac{\text{cubic feet}}{\text{minute}}$
Rate of walking to school	☐	☐	☐
Rate of painting a bedroom wall	☐	☐	☐
Rate of filling a bucket with water	☐	☐	☐
Rate of mopping the kitchen floor	☐	☐	☐

11. I would graph both equations on DESMOS and see where they intersect.

A system of equations is given.

$y = x^2 - 9$
$y = -2x - 1$

What is one solution to the system of equations?

(_____ , _____)

12. Emerson has \$120. Each week, he saves an additional \$15.

Write a function $f(x)$ that models the total amount of money Emerson has after x weeks.

$f(x) =$ _____